1. According to Piazza post [@1180](https://piazza.com/class/jzqwd6s59yh6bm?cid=1180), this means we have to prove , there existsa weighted vertex cover problem whose linear programming solution , where is the optimal (minimum) weight.

Let’s consider a graph with vertices. Let there be an edge between every pair of vertices in . Let the weight for each vertex. It’s obvious that the optimal weighted vertex cover is by choosing any vertices (the complement set of vertices of a graph’s vertex cover is an independent set, and all possible independent sets of such a graph as we described have cardinality because every pair of vertices is connected by an edge). The optimal linear programming solution has for every vertex, and so (it’s impossible for to be less than because then the average would be less than , and that cannot satisfy the constraint ).

If we graph (we put in the ceiling function because number of vertices is always a natural number), we see that starts from and asymptotically approaches . As approaches , is satisfied as approaches infinity. When , , so a smaller could satisfy . Choose an whose is less than . Thus, , there exists a weighted vertex cover problem whose linear programming solution is less than .

1. We formulate this into a linear programming problem.

Objective function to minimize:

Constraints:

After we obtain , round all to and all other ’s to . If and only if is .

**Proof of correctness**:

1. We want to show that this indeed produces a hitting set

This problem is essentially vertex cover on a hypergraph. An edge in a hypergraph has multiple vertices, and as long as one vertex on an edge is hit, the entire edge is covered. Our constraints ensure that at least one is greater than or equal to , which in turn is greater than or equal to because , and so that would be rounded up to to be included in .

1. We want to show this is a c-approximation.

The first inequality is because . The second inequality is because now it also includes . The third inequality is because , because found by our algorithm can be fractions while the solution found by integer programming is all integers. QED.

**Runtime Analysis**: we’re taught linear programming is **polynomial** time.